

Structure-Driven Turbulence in 'No Man's Land': A Bound for Fluctuation Amplitude in Drift Hole - Zonal Flow System

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Ads: This conference: EP session, Thur. Afternoon. (over...)
Paper: Y.K. + P.D. PoP '11, M.L. + P.D. '12 (submitted), Y.K. + P.D. '12 (submitted)
WCI Center web cite: Collection of talks, papers, etc

Outline

- Background and Motivation

→ Drive in No Man's Land: Incoming Holes?!

Eg) density blob/hole, **phase space density hole** (Vlasov + 3D magnetized)

→ Role of **zonal flow** in drift hole driven turbulence?

- Revisiting drift hole calculation with zonal flow coupling

→ **Structure**: **shift** of potential, **hole-flow resonance**

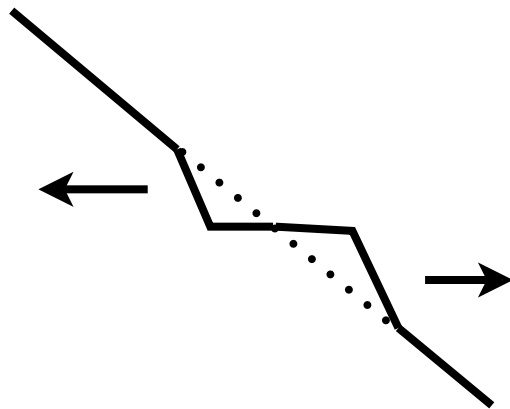
→ **Dynamics**: coupled DH-ZF system, self-regulating **predator-prey behavior**

- An application: amplitude bound in DH-ZF system

- Conclusion

Structure driven turbulence:

- **Structure** as drive for No Man's Land turbulence??
- **Real space structure:** Density blob/hole (Boedo '01, '03)

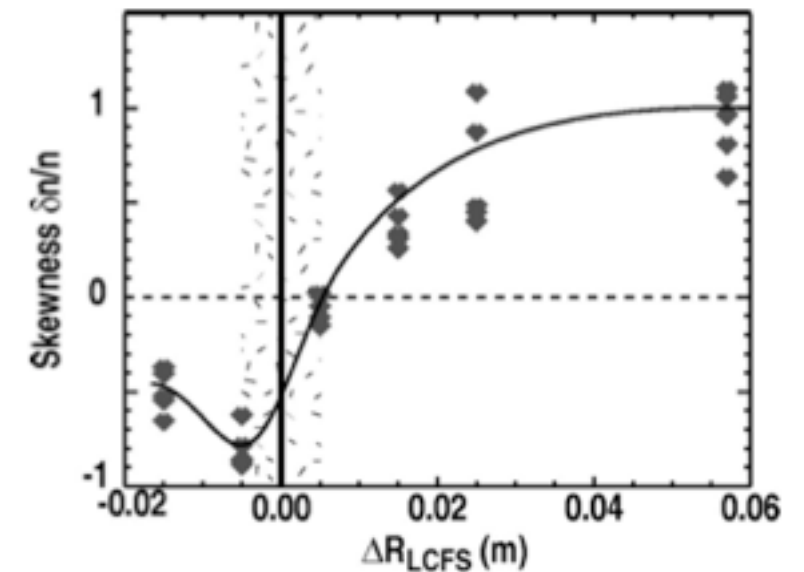


→ Edge turbulence

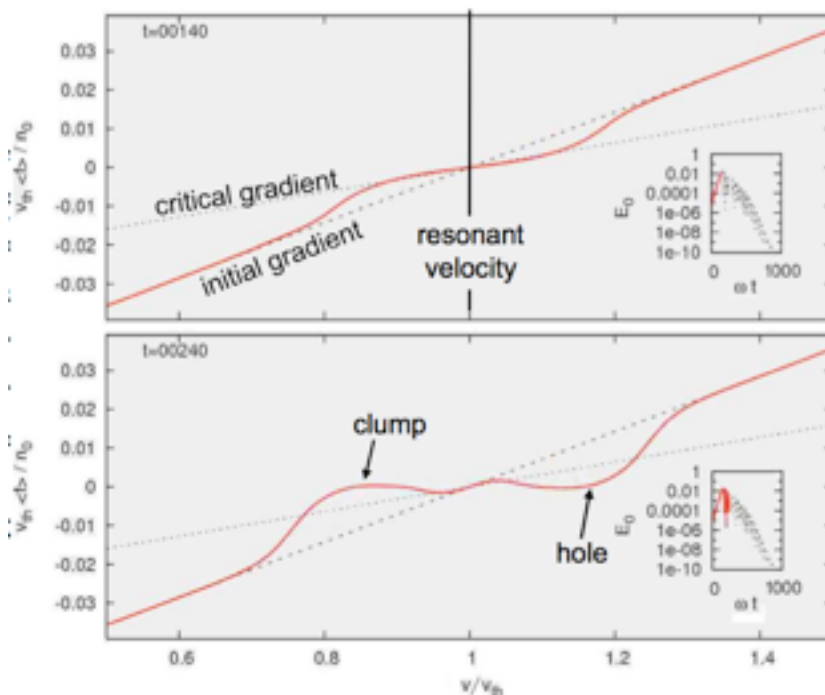
→ Blob ejection

→ Hole creation

→ Incoming hole as drive for turbulence in No Man's Land??



- Analogous to Hole-Clump (**phase space structures**, ala B+B) driven turbulence in EP problem



→ Resonance, Velocity gradient flattening

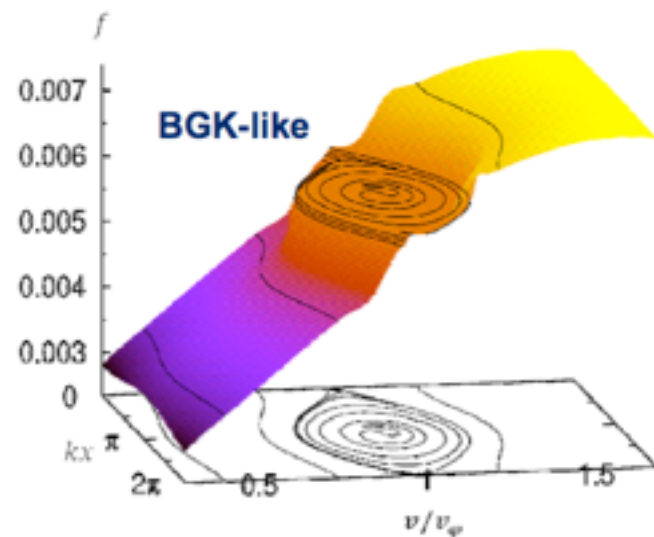
→ clump-hole pair creation

→ Subcritical Instability, observed numerically (B+B '97 Lesur '12, Lesur EP session, this meeting)

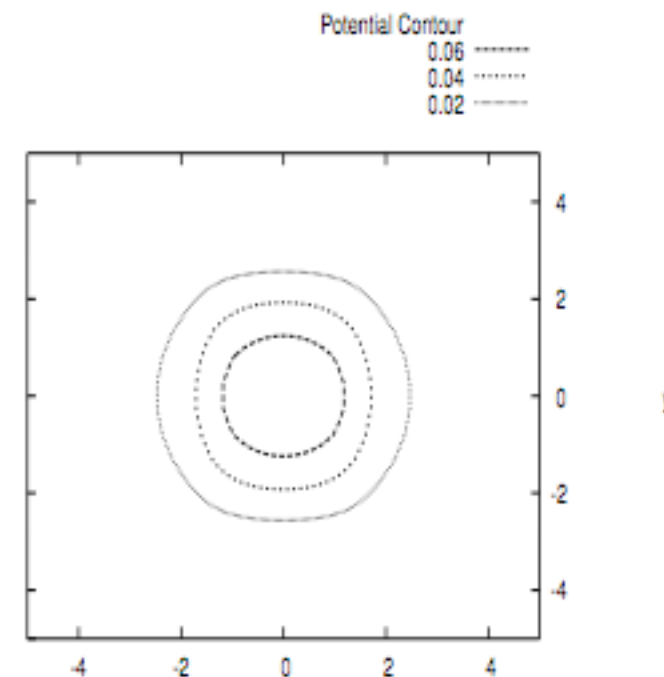
Structure driven turbulence:

- **Structure** as drive for No Man's Land turbulence??
- **Phase space structure:** Drift Hole (Terry + Diamond + Hahm '83)
 - BGK solution for inhomogeneous, 3D magnetized plasmas

parallel → akin to 1D

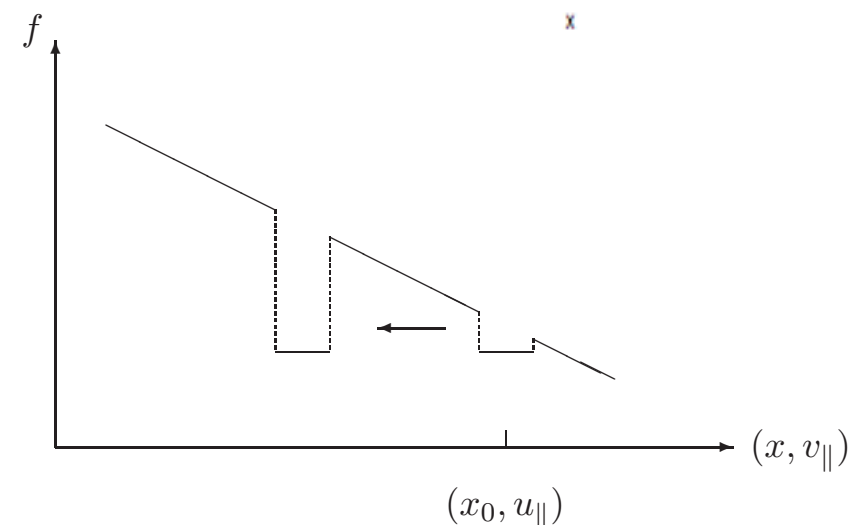


perpendicular → ExB vortex



- Taping of free energy:

- **Spatial** scattering of other species
(Quasi-neutral constraint)
- Displacement, Growth
- Can be subcritical!



Extension of drift hole paradigm:

- **Zonal flow coupling** in drift hole physics?

→ Impact on **saturation** of drift hole growth and **evaluation of saturation amplitude**

→ MUST include ZF effect for precise evaluation!

- How was it missed?

→ Naively, Reynolds stress via drift hole potential:

$$\langle \tilde{v}_x \tilde{v}_y \rangle = \text{Re} \sum_k i k_y \phi_{-k} \frac{1}{\lambda_k} \phi_k \rightarrow 0$$

- However: DH growth → Scattering of polarization charge → ZF coupling (P.D. '91,'08, Y.K. '11)

$$\langle \tilde{v}_x \nabla_{\perp}^2 \tilde{\phi} \rangle = \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle \neq 0 \quad \text{in general...}$$

➔ will show: **Hole-Flow resonance** provides proper phase, allows DH-ZF coupling!

➔ **revisiting drift hole problem with zonal flow coupling!**

Structure: dressing potential & screening length

- **Resonance/Trapping** of electron → Screening → **Dressing potential**

- **Dressing potential** → GK Poisson Eq. (Terry, Diamond, Hahm '83)

$$(\partial_x^2 - \lambda_k^{-2} [\langle v_y \rangle]) \frac{e\phi_k}{T_e} = \frac{1}{\rho_s^2} f_H \Delta v_{\parallel} \frac{\Delta y}{L_y} \frac{2}{k_y \Delta y} \sin \frac{k_y \Delta y}{2}$$

Trapped electrons

Screening charges: guiding center ions, polarization ions, adiabatic electrons

- **Localized** potential $\phi \sim e^{-x/\lambda_k}$ when $\lambda_k^2 > 0$ → otherwise ($\lambda_k^2 < 0$) $\phi \sim e^{ik_x x}$
 → Potential not-localized, Dressed potential radiated away

- **Flow** → Screening length:

χ : susceptibility

$$\lambda_k^{-2} \propto \chi(\mathbf{k}, k_{\parallel} u_{\parallel}, \langle v_y \rangle)$$

$$\cong \chi^{(0)}(\mathbf{k}, k_{\parallel} u_{\parallel}) - \left(\frac{k_y c_s}{k_{\parallel} u_{\parallel}} \right)^2 \frac{\langle v_y \rangle}{c_s} + i\omega_{*e} \pi \delta(k_{\parallel} u_{\parallel} - k_y \langle v_y \rangle)$$

Flow-Hole resonance → phase 'i'

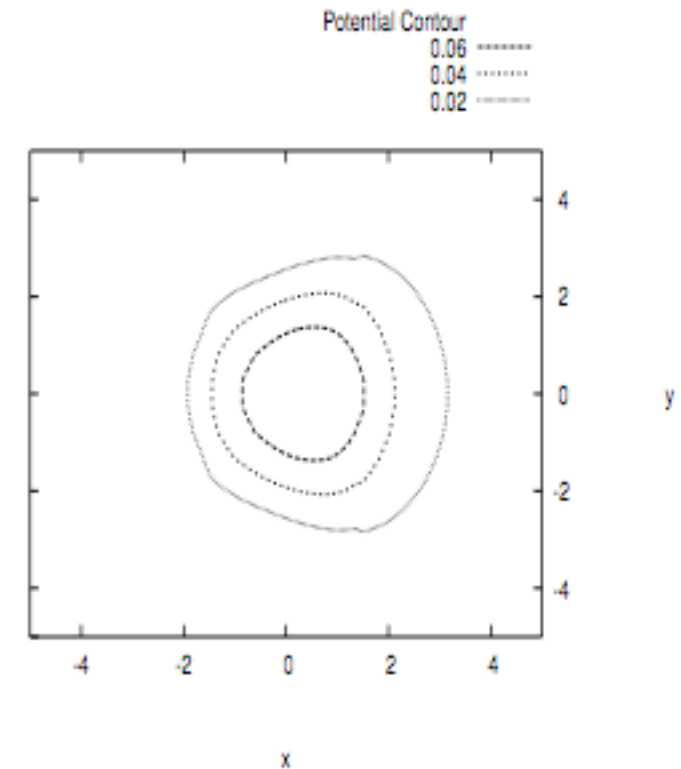
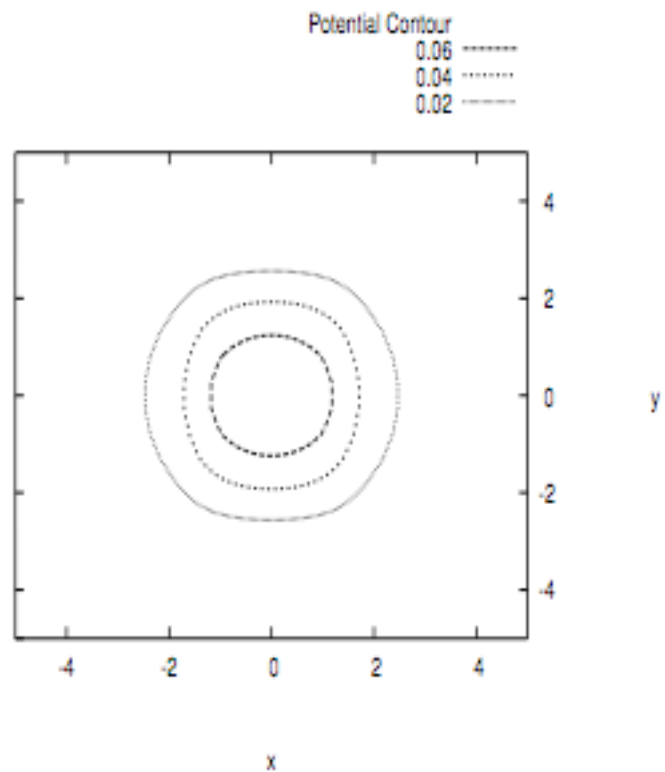
From **expansion**: different screening for $x > 0$ and $x < 0$, potential not screened symmetrically

Potential with Flow

- Dressing Potential: $\phi = \phi^{(0)} + \phi^{shift} + \phi^{res}$

- $\phi^{(0)}$:Up to Flow \rightarrow Isotropic ExB Vortex

- ϕ^{shift} : **Shift** to Isotropic ExB Vortex



\rightarrow Analogous to Shift of Drift Wave Eigenmode Structure by Shear Flow

- ϕ^{res} : **Resonance**

\rightarrow Non-Zero Reynolds Forcing:

\rightarrow Plausible: Resonance \rightarrow absorption
 \rightarrow zonal flow drive!

$$\langle \tilde{v}_x \tilde{v}_y \rangle = \frac{c^2}{B^2} \text{Re} \sum_k i k_y (\phi_{-k}^{(0)} + \phi_{-k}^{shift}) \partial_x \phi_k^{res} + \frac{c^2}{B^2} \text{Re} \sum_k i k_y \phi_{-k}^{res} \partial_x (\phi_k^{(0)} + \phi_k^{shift})$$

Dynamics:

- Non-Zero Reynolds Forcing → Drift Hole Potential dressed by Zonal Flow

- Zonal Flow Effects on **Dynamics?**

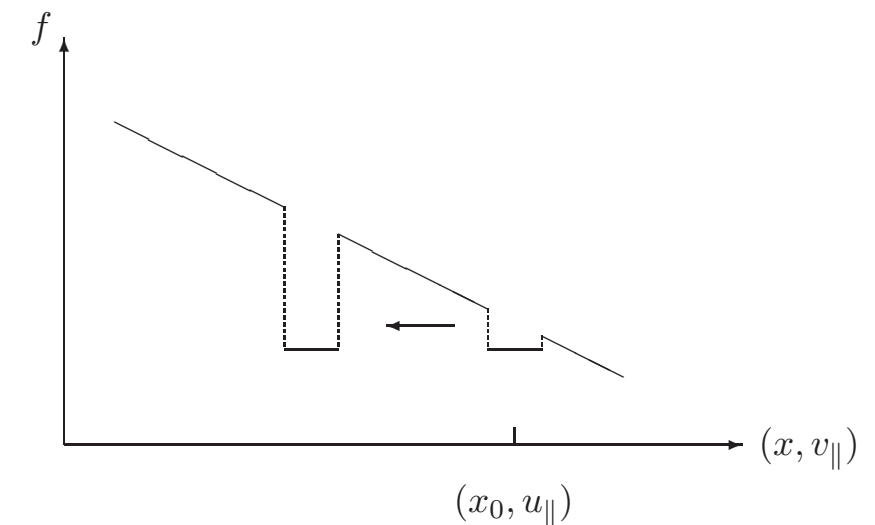
→ **Displacement** of phase space density blob

$$\frac{\partial}{\partial t} \int dv_{\parallel} \frac{\langle \delta f_e^2 \rangle}{2} = - \left\langle \tilde{v}_x \frac{\delta n_e}{n_0} \right\rangle \frac{\partial \langle f_e \rangle}{\partial x} \Big|_0 + \frac{|e|}{m_e} \left\langle \tilde{E}_{\parallel} \frac{\delta n_e}{n_0} \right\rangle \frac{\partial \langle f_e \rangle}{\partial v_{\parallel}} \Big|_0$$

→ Quasi-neutral constraint:

Electron DH must scatter ions, polarization charge

$$\delta n_e = \delta n_i^{GC} + n_0 \rho_s^2 \nabla_{\perp}^2 e \tilde{\phi} / T_e$$



→ Overall Growth:

$$\frac{\partial}{\partial t} \int dv_{\parallel} \frac{\langle \delta f_e^2 \rangle}{2} = - \langle f_e \rangle \Big|_0 \sum_{\mathbf{k}} (\omega_{*e} \Big|_0 - k_{\parallel} u_{\parallel}) \text{Im} \chi_i \left| \frac{e \tilde{\phi}}{T_e} \right|_{\mathbf{k}}^2$$

$$- \frac{1}{\omega_{ci}} \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle \frac{\partial \langle f_e \rangle}{\partial x} \Big|_0 - \frac{m_i}{m_e} \frac{c^2}{B^2} \partial_x \langle \tilde{E}_x \tilde{E}_{\parallel} \rangle \frac{\partial \langle f_e \rangle}{\partial v_{\parallel}} \Big|_0$$

Guiding center ions

Polarization ions

→ zonal flow + toroidal flow
(McDevitt '09)

→ couples to intrinsic rotation

Subcritical Instability

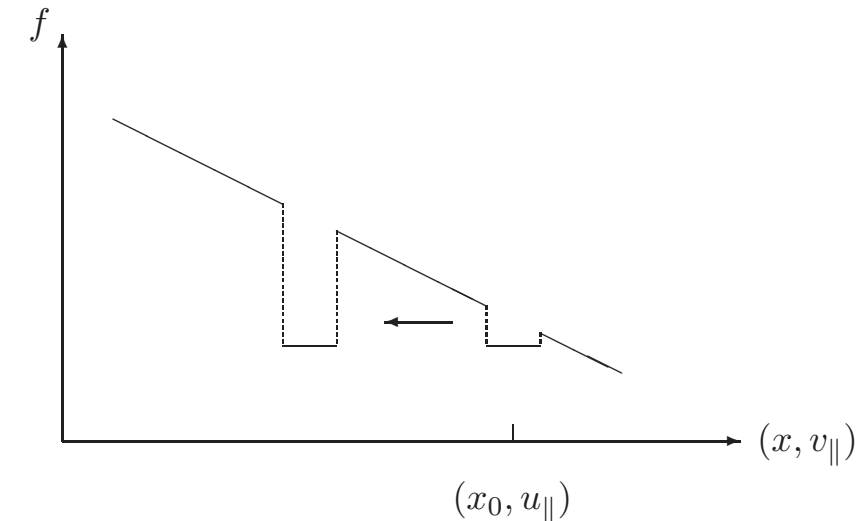
- Up to Flow → `Bare' Growth of Drift Hole, **Subcritical Instability**

$$\gamma \sim \langle f_e \rangle|_0 \Delta v_{\parallel} (\omega_{*e}|_0 - k_{\parallel} u_{\parallel}) \frac{|\text{Im}\chi_i|}{|\chi|^2} \sim k_{\parallel} \Delta v_{\parallel} \frac{|\text{Im}\chi_e \text{Im}\chi_i|}{|\chi|^2}$$

→ **Free Energy** Required: $\omega_{*e}|_0 > k_{\parallel} u_{\parallel}$

→ **Nonlinear**: $\Delta v_{\parallel} \propto \sqrt{\phi}$

→ **Ion Dissipation as Drive**: Scattering of Ions Required for Growth



- Comparison to **Linear Drift Wave** Growth:

	Electron drift wave instability	Electron drift hole instability
Growth rate	$\gamma_L \sim \partial\chi/\partial\omega_k ^{-1} (\text{Im}\chi_e - \text{Im}\chi_i)$	$\gamma \sim k_{\parallel} \Delta v_{\parallel} \text{Im}\chi_e \text{Im}\chi_i / \chi ^2$
Access to free energy	$\omega_* > \omega$	$\omega_{*e} _0 > k_{\parallel} u_{\parallel}$
Amplitude dependence	No	Yes, $\propto \Delta v_{\parallel} \sim \sqrt{\phi}$
Ion Landau Damping	Stabilizing	Destabilizing
Type of instability	Supercritical, exponential growth	Subcritical, explosive growth

TABLE I. Comparison between linear and nonlinear instabilities.

Coupled Evolution: Closed Feedback Loop

- Drift Hole Dynamics with Zonal Flow → **Coupled Evolution, Self-Regulating Behavior**

$$\frac{\partial}{\partial t} \int dv_{\parallel} \frac{\langle \delta f_e^2 \rangle}{2} = - \langle f_e \rangle|_0 \sum_{\mathbf{k}} (\omega_{*e}|_0 - k_{\parallel} u_{\parallel}) \text{Im} \chi_i \left| \frac{e\tilde{\phi}}{T_e} \right|_{\mathbf{k}}^2$$

$$\rightarrow \frac{\partial}{\partial t} \left(\int dv_{\parallel} \frac{\omega_{ci} \langle \delta f_e^2 \rangle}{2 \partial \langle f_e \rangle / \partial x|_0} - \langle v_y \rangle \right) = \frac{c_s^2}{v_*} \sum_{\mathbf{k}} (\omega_{*e}|_0 - k_{\parallel} u_{\parallel}) \text{Im} \chi_i \left| \frac{e\tilde{\phi}}{T_e} \right|_{\mathbf{k}}^2 + \nu_d^{\perp} \langle v_y \rangle$$

→ **Momentum** budget for flow and turbulence pseudomomentum

→ akin to **Momentum Theorems** by Charney & Drazin:

$$\int dv_{\parallel} \langle \delta f_e^2 \rangle / \langle f_e \rangle' |_0$$

→ kinetic extension of pseudomomentum

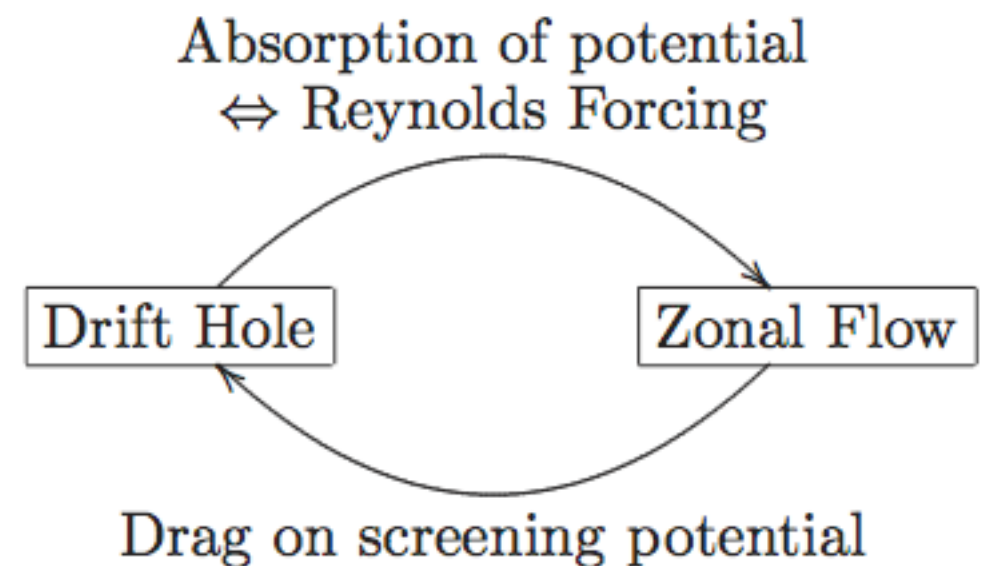
→ Self-Regulating **Predator-Prey** Behavior

Prey: Drift Hole

Predator: Zonal Flow

→ Analogous to self-regulation of DW-ZF with **energy** conservation

$$\partial_t (\mathcal{E}_{DW} + V'_{ZF}{}^2) = \gamma \mathcal{E}_{DW} - \nu V'_{ZF}$$



Saturation

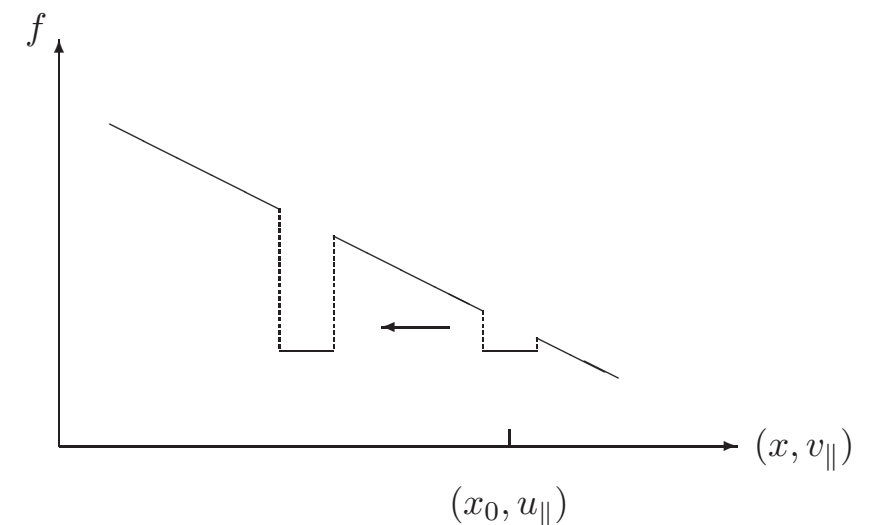
- Coupled Drift Hole+Zonal Flow Evolution → Saturation with Flows

$$\frac{\langle v_y \rangle}{c_s} = -\frac{1}{\nu_d^\perp} \frac{c_s}{v_*} \sum_{\mathbf{k}} (\omega_{*e}|_0 - k_{\parallel} u_{\parallel}) \text{Im} \chi_i \left| \frac{e\tilde{\phi}}{T_e} \right|_{\mathbf{k}}^2$$

→ Stationary State with Non-Zero Flow: Akin to Enhanced State in Predator-Prey

→ Hole Mixing ceases via Turbulent Drag from Flow

→ Zonal Flow Driven by a single coherent phase space structure: Does not Require Inverse Cascade, Wave based Mechanisms (Coherent, W.K.E.) etc.



An Application: Bound on Amplitude

- Non-Zero Flow Results at Stationary State → Feedback to Structure itself? i.e. $\lambda[\langle v_y \rangle]$
- Screening must be positive **even with flows** for **localized dressing potential to form**
→ Flows may undress DH potential...

$$\lambda^{-2} \propto \chi[\langle v_y \rangle] > 0 \quad \Rightarrow \quad \frac{\langle v_y \rangle}{c_s} < \min \left(\frac{u_{\parallel}}{c_s} \frac{k_{\parallel}}{k_y} \frac{\chi^{(0)}}{1 + k_y^2 \rho_s^2} \right)$$

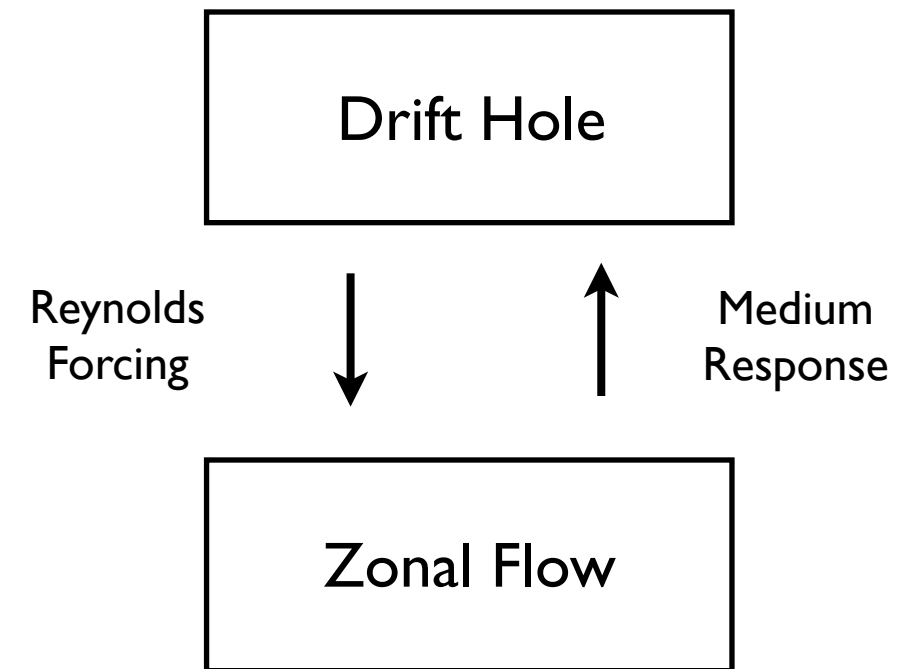
- Turbulent driven zonal flow must satisfy the condition
- Bound on Amplitude

$$\left| \frac{e\tilde{\phi}}{T_e} \right|^2 < \frac{\nu_d^{\perp}}{\omega_{ci}} \frac{1}{\rho_s k_y \text{Im}\chi_i} \min \left(\frac{u_{\parallel}}{c_s} \frac{k_{\parallel}}{k_y} \frac{\chi^{(0)}}{1 + k_y^2 \rho_s^2} \right)$$

- ZF damping as controlling parameter: akin to Predator-Prey
- increases toward edge

Conclusion

- Drive in No Man's Land turbulence: Incoming **Drift Hole**?
- Drift Hole coupled to Zonal Flow
 - made possible with **flow-hole resonance**
 - **self-regulating** behavior of DH-ZF system
 - Amplitude bound in DH-ZF system: DH effects become increasingly important toward edge
- Other mechanisms: Real space density hole?
Granulation (multiple structures)?



Bird's Eye View:

Turbulence in No man's Land

Local linear instability

→ not robust

Else

→ others ?

Density Blob/Hole

Incoming Structures
→ Tap free energy?

Edge Turbulence

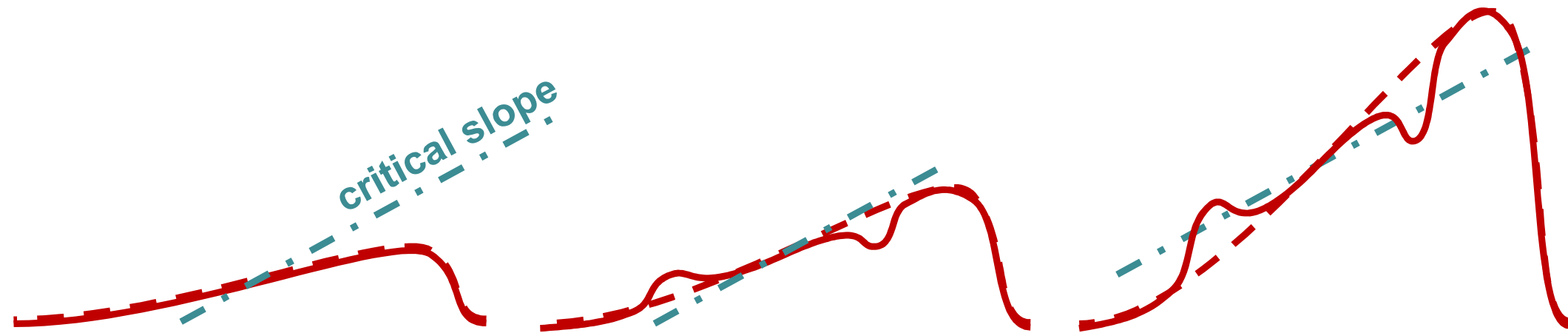
Drift Hole
(single structure)

Drift Granulation
(multi-structures)

phase space structures

⚠ Subcritical instability

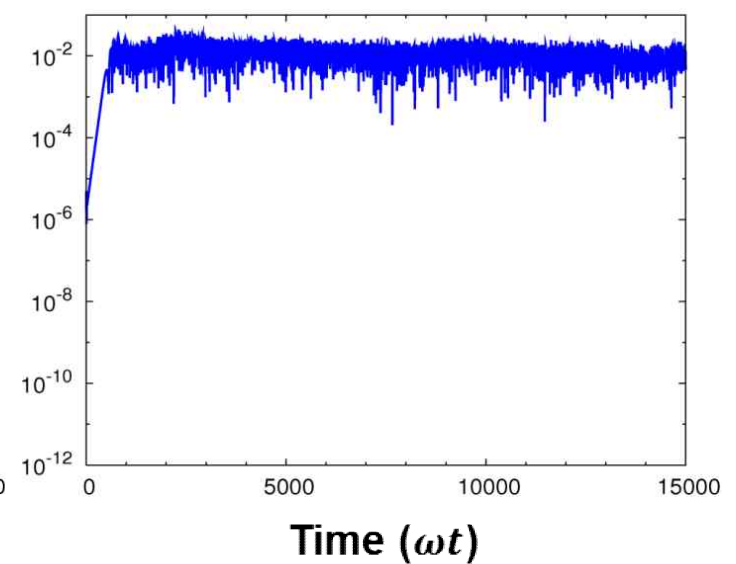
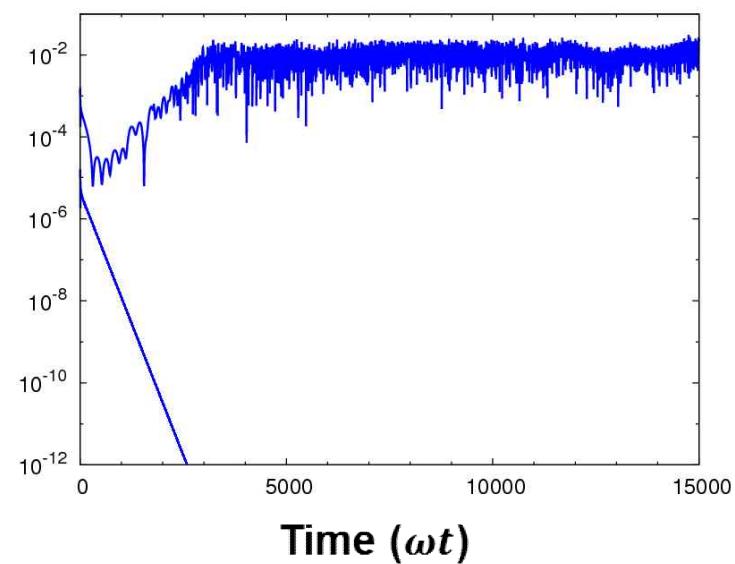
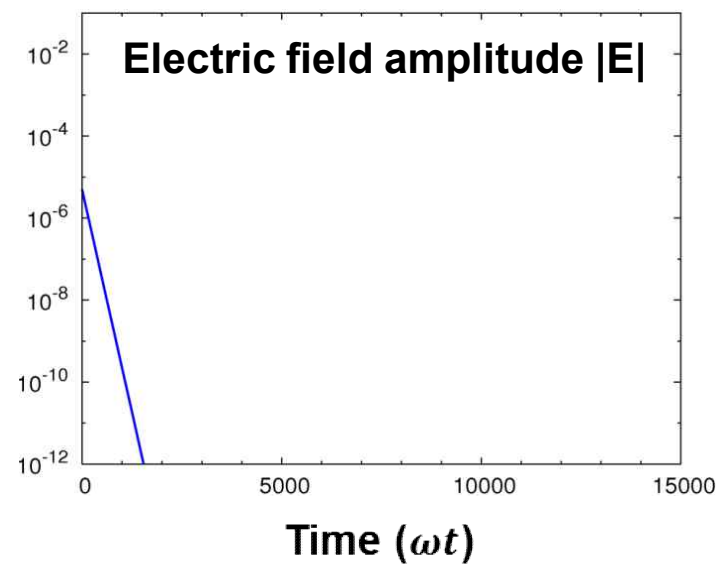
Linear growth rate $\gamma \approx \gamma_L - \gamma_d \Rightarrow$ Critical slope $\gamma_L = \gamma_d$



Stable, $\gamma < 0$

**Nonlinearly unstable, $\gamma < 0$
(Subcritical instability)**

Unstable, $\gamma > 0$



Zonal Flow Coupling

- Flux of polarization charge:

$$\langle \tilde{v}_x \nabla_{\perp}^2 \tilde{\phi} \rangle = \frac{c}{B} \text{Re} \sum_k i k_y (\phi_{-k}^{(0)} + \phi_{-k}^{shift} + \phi_{-k}^{res}) \partial_x^2 (\phi_k^{(0)} + \phi_k^{shift} + \phi_k^{res})$$

- Phase: $\phi^{(0)}$ ϕ^{shift} pure real ϕ^{res} pure imaginary

- Non-zero polarization charge flux

$$\langle \tilde{v}_x \nabla_{\perp}^2 \tilde{\phi} \rangle = \frac{c}{B} \text{Re} \sum_k i k_y \{ (\phi_{-k}^{(0)} + \phi_{-k}^{shift}) \partial_x^2 \phi_k^{res} + \phi_{-k}^{res} \partial_x^2 (\phi_k^{(0)} + \phi_k^{shift}) \} \neq 0$$

→ Non-zero Reynolds Forcing, ZF coupling

$$\therefore \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle \propto \langle \tilde{v}_x \nabla_{\perp}^2 \tilde{\phi} \rangle \neq 0$$